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On e-$\mathcal{I}$-open sets, e-$\mathcal{I}$-continuous functions and decomposition of continuity

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Abstract: In this paper, we introduce the notations of e-$\mathcal{I}$-open sets and strong $\mathcal{B}_I^*$-set to obtain a decomposition of continuing via idealization. Additionally, we investigate properties of e-$\mathcal{I}$-open sets and strong $\mathcal{B}_I^*$-set. Also we studied some more properties of e-$\mathcal{I}$-open sets and obtained several characterizations of e-$\mathcal{I}$-continuous functions and investigate their relationship with other types of functions.

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1 Introduction and Preliminaries

The subject of ideals in topological spaces has been studied by Kuratowski [12] and Vaidyanathaswamy [25]. Jankovic and Hamlett [11] investigated further properties of ideal space. The importance of continuity and generalized continuity is significant in various areas of mathematics and related sciences. One of them, which has been in recent years of interest to general topologists, is its decomposition. The decomposition of continuity has been studied by many authors. The class of e-open sets is contains all δ-preopen [15] sets and δ-semiopen [14] sets. In this paper, we introduce the notation of e-$\mathcal{I}$-open sets which is a generalization of $semi^*$-$\mathcal{I}$-open sets [8] and pre^*-$\mathcal{I}$-open [5] sets is introduced, and strong $\mathcal{B}_I^*$-set to obtain a decomposition of continuing via idealization. Additionally, we investigate properties of e-$\mathcal{I}$-open sets and strong $\mathcal{B}_I^*$-set. Also we studied some more properties of e-$\mathcal{I}$-open sets and obtained several characterizations of e-$\mathcal{I}$-continuous functions and investigate their relationship with other types of functions.
A subset A of a space (X, τ) is said to be regular open (resp. regular closed) [23] if $A = Int(Cl(A))$ $(resp. A = Cl(Int(A)))$. A is called δ-open [26] if for each $x \in A$,

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there exist a regular open set G such that $x \in G \subset A$. The complement of δ-open set is called δ-closed. A point $x \in X$ is called a δ-cluster point of A if $Int(Cl(U)) \cap A \neq \emptyset$ for each open set U containing x. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by $Cl_\delta(A)$ [26]. The set δ-interior of A [26] is the union of all regular open sets of X contained in A and its denoted by $Int_\delta(A)$. A is δ-open if $Int_\delta(A) = A$. The collection of all δ-open sets of (X, τ) is denoted by $\delta O(X)$ and forms a topology τ^δ. The topology τ^δ is called the semi regularization of τ and is denoted by τ_s.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the following conditions:
$A \in \mathcal{I}$ *and* $B \subset A$ *implies* $B \in \mathcal{I}$; $A \in \mathcal{I}$ *and* $B \in \mathcal{I}$ *implies* $A \cup B \in \mathcal{I}$. Applications to various fields were further investigated by Jankovic and Hamlett [11] Dontchev et al. [3]; Mukherjee et al. [13]; Arenas et al. [2]; et al. Nasef and Mahmoud [18], etc. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^* : \wp(X) \to \wp(X)$, called a local function [24, 11] of A with respect to τ and $\mathcal{I}$ is defined as follows: for $A \subseteq X$,

$$A^*(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I} \text{ for every U} \in \tau(x)\}$$

where $\tau(x) = \{U \in \tau \mid x \in U\}$. A Kuratowski closure operator $Cl^*(x) = A \cup A^*(\mathcal{I}, \tau)$. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{I}, \tau)$. X^* is often a proper subset of X.

A subset A of an ideal space (X, τ) is said to be *R-I*-open (resp. *R-I*-closed) [28] if $A = Int(Cl^*(A))$ $(resp. A = Cl^*(Int(A))$. A point $x \in X$ is called $\delta - I$-cluster point of A if $Int(Cl^*(U)) \cap A \neq \emptyset$ for each open set U containing x. The family of all δ-$\mathcal{I}$-cluster points of A is called the δ-$\mathcal{I}$-closure of A and is denoted by $\delta Cl_I(A)$. The set δ-$\mathcal{I}$-interior of A is the union of all *R-I*-open sets of X contained in A and its denoted by $\delta Int_I(A)$. A is said to be δ-$\mathcal{I}$-closed if $\delta Cl_I(A) = A$ [28].

Definition 1.1. *A subset A of a topological space X is called*

1. *β-open [1] if $A \subset Cl(Int(Cl(A)))$.*
2. *α-open [19] if $A \subset Int(Cl(Int(A)))$.*
3. *t-set [22] if $Int(A) = Int(Cl(A))$.*
4. *e-open set [7] if $A \subset Int(\delta Cl(A)) \cup Cl(\delta Int(A))$.*
5. *strongly B-set [7] if $A = U \cap V$ where U is an open set and V is a t-set and $Int(Cl(A)) = Cl(Int(A))$.*
6. *δ-preopen [15] if $A \subset Int(\delta Cl(A))$.*
7. *δ-semiopen [14] if $A \subset Cl(\delta Int(A))$.*
8. *a-open [4] if $A \subset Int(Cl(\delta Int(A)))$.*

The class of all δ-preopen (resp. δ-semiopen, a-open) sets of (X, τ) is denoted by $\delta PO(X)$ (resp. $\delta SO(X)$, $aO(X)$).

Definition 1.2. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called*

1. *$\delta\alpha$-$\mathcal{I}$-open [8] if $A \subset Int(Cl(\delta Int_I(A)))$.*
2. *semi*-$\mathcal{I}$-open [8] if $A \subset Cl(\delta Int_I(A))$.*
3. *pre*-$\mathcal{I}$-open [5] if $A \subseteq Int(\delta Cl_I(A))$.*
4. *Strongly t-$\mathcal{I}$-set [5] if $Int(A) = Int(\delta Cl_I(A))$.*
5. *Strongly B-$\mathcal{I}$-set [5] if $A = U \cap V$ where U is an open set and V is a Strongly t-I-set.*
6. *$\delta\beta_I$-open [8] if $A \subset Int(Cl(\delta Int_I(A)))$.*
7. *B_I-set [9] if $A = U \cap V$ where U is an open set and V is a t-I-set.*

The class of all *semi**-$\mathcal{I}$-open (resp. *pre**-$\mathcal{I}$-open, $\delta\beta_I$-open, $\delta\alpha$-$\mathcal{I}$-open) sets of $(X, \tau, \mathcal{I})$ is denoted by $S^*IO(X)$ (resp. $P^*IO(X)$, $\delta\beta IO(X)$, $\delta\alpha IO(X)$). [8, 5].

2 e-$\mathcal{I}$-open

Definition 2.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be e-$\mathcal{I}$-open if $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$.*

The class of all e-$\mathcal{I}$-open sets in X will be denoted by $EIO(X, \tau)$.

Proposition 2.2. *Let A be an e-$\mathcal{I}$-open such that $\delta Int_I(A) = \emptyset$, then A is pre*-$\mathcal{I}$-open. For a subset of an ideal topological space the following hold:*

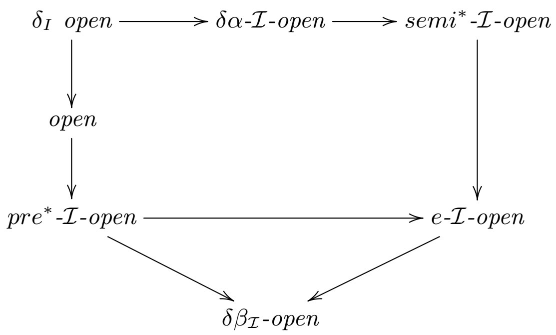
1. *Every semi*-$\mathcal{I}$-open is e-$\mathcal{I}$-open,*
2. *Every pre*-$\mathcal{I}$-open is e-$\mathcal{I}$-open,*
3. *Every e-$\mathcal{I}$-open is $\delta\beta_I$-open.*

Proof. (1) Obvious.
(2) Obvious.
(3) Let A be e-$\mathcal{I}$-open. Then we have

$$\begin{aligned} A &\subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) \\ &\subset Cl(Int(\delta Int_I(A))) \cup Int(Int(\delta Cl_I(A))) \\ &\subset Cl(Int(\delta Int_I(A)) \cup Int(\delta Cl_I(A))) \\ &\subset Cl[Int(\delta Int_I(A)) \cup \delta Cl_I(A)] \\ &\subset Cl[Int(\delta Cl_I(A \cup A))] \\ &= Cl(Int(\delta Cl_I(A))). \end{aligned}$$

This show that A is an $\delta\beta_I$-open set. □

Remark 2.3. *From above the following implication and none of these implications is reversible as shown by examples given below*



Example 2.4. *Let* $X = \{a, b, c, d\}$ *with a topology* $\tau = \{\emptyset, X, \{b\}, \{a, d\}, \{a, b, d\}\}$ *and an ideal* $\mathcal{I} = \{\emptyset, \{b\}\}$. *Then the set* $A = \{b, d\}$ *is e-*$\mathcal{I}$*-open, but is not semi***-*$\mathcal{I}$*-open. Because* $Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) = Cl(\emptyset) \cup Int(X) = \emptyset \cup X = X \supset A$ *and hence* A *is e-*$\mathcal{I}$*-open. Since* $Cl(\delta Int_I(A)) = Cl(\emptyset) = \emptyset \not\supseteq A$. *So* A *is not semi***-*$\mathcal{I}$*-open.*

Example 2.5. *Let* $X = \{a, b, c\}$ *with a topology* $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ *and an ideal* $\mathcal{I} = \{\emptyset, \{b\}\}$. *Then the set* $A = \{a, c\}$ *is e-*$\mathcal{I}$*-open, but is not pre***-*$\mathcal{I}$*-open. For* $Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) = Cl(\{a, b\}) \cup Int(\{a, c\}) = \{a, b, c\} \cup \{a\} = X \supset A$ *and hence* A *is e-*$\mathcal{I}$*-open. Since* $Int(\delta Cl_I(A)) = Int(\{a, c\}) = \{a\} \not\supseteq A$. *Hence* A *is not Pre***-*$\mathcal{I}$*-open.*

Example 2.6. *Let* $X = \{a, b, c, d\}$ *with a topology* $\tau = \{\emptyset, X, \{b\}, \{a, d\}, \{a, b, d\}\}$ *and an ideal* $\mathcal{I} = \{\emptyset, \{b\}\}$. *Then the set* $A = \{a, c\}$ *is* $\delta\beta_{\mathcal{I}}$*-open, but is not e-*$\mathcal{I}$*-open. Since* $Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) = Cl(\emptyset) \cup Int(\{a, c, d\}) = \{a, d\} \not\supseteq A$ *and hence* A *is not e-*$\mathcal{I}$*-open. For* $Cl(Int(\delta Cl_I(A))) = Cl(Int(\{a, c, d\})) = Cl(\{a, d\}) = \{a, c, d\} \supseteq A$. *Hence* A *is* $\delta\beta_{\mathcal{I}}$*-open.*

Proposition 2.7. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and let* A, $U \subseteq X$. *If* A *is e-*$\mathcal{I}$*-open set and* $U \in \tau$. *Then* $A \cap U$ *is an e-*$\mathcal{I}$*-open.*

Proof. By assumption $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$ and $U \subseteq Int(U)$. Then

$$\begin{aligned} A \cap U \subset & (Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))) \cap Int(U) \\ & \subset (Cl(\delta Int_I(A)) \cap Int(U)) \cup (Int(\delta Cl_I(A)) \cap Int(U)) \\ & \subset (Cl(\delta Int_I(A)) \cap Cl(Int(U))) \cup (Int(\delta Cl_I(A)) \cap Cl(Int(U))) \\ & \subset (Cl(\delta Int_I(A)) \cap Int(U)) \cup (Int(Cl(\delta Cl_I(A)) \cap Cl(Cl(Int(U))))) \\ & \subset Cl(\delta Int_I(A \cap U) \cup (Int(Cl(\delta Cl_I(A)) \cap Cl(Int(U)))) \\ & \subset Cl(\delta Int_I(A \cap U)) \cup (Int(Cl(\delta Cl_I(A)) \cap Int(U))) \\ & \subset Cl(\delta Int_I(A \cap U)) \cup (Int(\delta Cl_I(A \cap U))). \end{aligned}$$

Thus $A \cap U$ is e-$\mathcal{I}$-open. ☐

Definition 2.8. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be e-$\mathcal{I}$-closed if its complement is e-$\mathcal{I}$-open.*

Theorem 2.9. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is e-$\mathcal{I}$-closed, then $Cl(\delta Int_I(A)) \cap Int(\delta Cl_I(A)) \subset A$.*

Proof. Since A is e-$\mathcal{I}$-closed, $X - A$ is e-$\mathcal{I}$-open, from the fact τ^* finer than τ, and the fact $\tau^\delta \subset \tau^{\delta\mathcal{I}}$ we have,

$$\begin{aligned} X - A &\subset Cl(\delta Int_I(X - A)) \cup Int(\delta Cl_I(X - A)) \\ &\subset Cl(\delta Int(X - A)) \cup Int(\delta Cl(X - A)) \\ &= [X - [Cl(\delta Int(A))]] \cup [X - [Int(\delta Cl(A))]] \\ &\subset [X - [Cl(\delta Int_I(A))]] \cup [X - [Int(\delta Cl_I(A))]] \\ &= X - [[Cl(\delta Int_I(A))] \cap [Int(\delta Cl_I(A))]]. \end{aligned}$$

Therefore we obtain $[Cl(\delta Int_I(A)) \cap Int(\delta Cl_I(A))] \subset A$. □

Corollary 2.10. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ such that $X - [Cl(\delta Int_I(A))] = Int(\delta Cl_I(X - A))$ and $X - [Int(\delta Cl_I(A))] = Cl(\delta Int_I(X - A))$. Then A is e-$\mathcal{I}$-closed if and only if $[Cl(\delta Int_I(A)) \cap Int(\delta Cl_I(A))] \subset A$.*

Proof. Necessity: This is immediate consequence of Theorem 2.9
Sufficiency: Let $[Cl(\delta Int_I(A)) \cap Int(\delta Cl_I(A))] \subset A$. Then

$$\begin{aligned} X - A &\subset X - [Cl(\delta Int_I(A)) \cap Int(\delta Cl_I(A))] \\ &\subset [X - [Cl(\delta Int_I(A))]] \cup [X - [Int(\delta Cl_I(A))]] \\ &= Cl(\delta Int_I(X - A)) \cup Int(\delta Cl_I(X - A)) \end{aligned}$$

Thus $X - A$ is e-$\mathcal{I}$-open and hence A is e-$\mathcal{I}$-closed. □

If $(X, \tau, \mathcal{I})$ is an ideal topological space and A is a subset of X, we denote by $\mathcal{I}|_A$. If $(X, \tau, \mathcal{I})$ relative ideal on A and $\mathcal{I}|_A = \{A \cap I : I \in \mathcal{I}\}$ is obviously an ideal on A.

Lemma 2.11. *[11] Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B subsets of X such that $B \subset A$. Then $B^*(\tau|_A, \mathcal{I}|_A) = B^*(\tau, \mathcal{I}) \cap A$.*

Proposition 2.12. *Let $(X, \tau, \mathcal{I})$ be ideal topological space and let A, $U \subseteq X$. If A is an e-$\mathcal{I}$-open set and $U \in \tau$. Then $A \cap U \in EIO(U, \tau|_U, \mathcal{I}|_U)$.*

Proof. Straight forward from Proposition 2.7 □

Theorem 2.13. *If $A \in EIO(X, \tau, \mathcal{I})$ and $B \subset \tau$, then $A \cap B \in EIO(X, \tau, \mathcal{I})$.*

Proof. Let $A \in EIO(X, \tau, \mathcal{I})$ and $B \subset \tau$ then $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$ and

$$\begin{aligned} A \cap B &\subset [Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))] \cap B \\ &\subset [Cl(\delta Int_I(A)) \cap B] \cup [Int(\delta Cl_I(A)) \cap B] \\ &\subset [Cl(\delta Int_I(A \cap B))] \cup [Int(\delta Cl_I(A \cap B))]. \end{aligned}$$

This proof come from the fact $\delta Int_I(A)$ is the union of all R-$\mathcal{I}$-open of X contained in A. Then

$$\begin{aligned} A = Int(Cl^*(A)) \Rightarrow A \cap B &= Int(Cl^*(A)) \cap B \\ &= Int(A^* \cup A) \cap B \\ &= Int[(A \cap B) \cup (A^* \cap B)] \\ &\subset Int[Cl^*(A \cap B)] = A \cap B \end{aligned}$$

Hence $Cl(\delta Int_I(A)) \cap B \subset Cl(\delta Int_I(A \cap B))$, and other part is obvious. □

Proposition 2.14. *for any ideal topological space $(X, \tau, \mathcal{I})$ and $A \subset X$ we have:*

1. *If $I = \emptyset$, then A is e-$\mathcal{I}$-open if and only if A is e-open.*
2. *If $I = \wp(X)$, then A is e-$\mathcal{I}$-open if and only if $A \in \tau$.*
3. *If $I = N$, then A is e-$\mathcal{I}$-open if and only if A is e-open.*

Proof. (1) Let $I = \emptyset$ and $A \subset X$. We have $\delta Cl_I(A)) = \delta Cl(A))$, $\delta Int_I(A)) = \delta Int(A))$ and $A^* = Cl(A)$. on other hand, $Cl^*(A) = A^* \cup A = Cl(A)$. Hence $A^* = Cl(A) = Cl^*(A)$. Since A is e-$\mathcal{I}$-open

$$A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) = Cl(\delta Int(A)) \cup Int(\delta Cl(A))$$

Thus, A is e-open.
Conversely, let A is e-open. Since $I = \emptyset$, then

$$A \subset Cl(\delta Int(A)) \cup Int(\delta Cl(A)) = Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$$

Thus A is e-$\mathcal{I}$-open.
(2) Let $I = P(X)$ and $A \subset X$. We have $A^* = \emptyset$. Since $\delta Int_I(A))$ is the union of all *R-$\mathcal{I}$-open* contained in A, since $A^* = \emptyset$, then $Int(A) = A$, and $\delta Cl_I(A)$ is the family of all δ-$\mathcal{I}$- cluster points of A, since $A^* = \emptyset$, then $Int(A) \cap A \neq \emptyset$ On other hand

$$\begin{aligned} A &\subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) \\ &= Cl(Int(A)) \cup Int(Cl(A)) \\ &\subset Int(Cl(Int(A))) \cup Int(Cl(A)) \\ &= Int(Cl(Int(A)) \cup Int(Cl(A))) \\ &\subset Int(Cl(Int(A) \cup Cl(A))) \\ &\subset Int(Cl(Cl(A \cup A) \\ &\subset Int(Cl(A \cup A) = Int(Cl(A)). \end{aligned}$$

This show $A \in \tau$.
Conversely, It is shown in Remark 2.3 .
(3) Every e-$\mathcal{I}$-open is e-open.
Let A be e-$\mathcal{I}$-open then, $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$. by using this fact $A^* = Cl(A) = Cl^*(A)$, we have $\delta Cl_I(A) = \delta Cl(A)$, $\delta Int_I(A) = \delta Int(A)$, since $\delta Cl_I(A)$ is the familly of all δ-$\mathcal{I}$-cluster point of A, and $\delta Int_I(A)$ the union of all R-$\mathcal{I}$-open set of X we have respectively,

$$\begin{aligned} \emptyset \neq Int(Cl^*(U)) \cap A &= Int(U^* \cup U) \cap A = Int(Cl(U) \cup U) \cap A \\ &= Int(Cl(U)) \cap A \neq \emptyset \end{aligned}$$

From this we get $\delta Cl_I(A) = \delta Cl(A)$, and

$$\begin{aligned} A = Int(Cl^*(A)) &= Int(A^* \cup A) = Int[Cl(A) \cup A] \\ &= Int(Cl(A)) = A \end{aligned}$$

From this we get $\delta Int_I(A) = \delta Int(A)$. This show that

$$A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) \subset Cl(\delta Int(A)) \cup Int(\delta Cl(A))$$

Hence (3) is proved
Let us consider $I = N$ and A is e-open
If $I = N$ then $A^* = Cl^*(Int(Cl^* A))$.
Since A is e-open then $A \subset Cl(\delta Int(A)) \cup Int(\delta Cl(A))$. Then

$$\begin{aligned} \emptyset \neq Int(Cl(U)) \cap A &= Int(U \cup U) \cap A = Int(Cl(Int(Cl(U)) \cup U) \cap A \\ &\subset Int(Cl^*(Int(Cl^*(U))) \cup U) \cap A = Int(U^* \cup U) \cap A = Int(Cl^*(U)) \cap A \neq \emptyset \end{aligned}$$

From this we get $\delta Cl(A) \subset \delta Cl_I(A)$, and

$$\begin{aligned} A = Int(Cl(A)) &= Int(A \cup A) = Int[Cl(Int(Cl(A))) \cup A] \\ &\subset Int[Cl^*(Int(Cl^*(A))) \cup A] = Int(A^* \cup A) = Int(Cl^*(A)) = A \end{aligned}$$

From this we get $\delta Int(A) \subset \delta Int_I(A)$.
A is e-$\mathcal{I}$-open. Hence the proof. □

Proposition 2.15. 1. *The union of any family of e-$\mathcal{I}$-open sets is an e-$\mathcal{I}$-open set.*

2. *The intersection of even two e-$\mathcal{I}$-open open sets need not to be e-$\mathcal{I}$-open as shown in the following example.*

Proof. (1) Let $\{A_\alpha / \alpha \in \Delta\}$ be a family of e-$\mathcal{I}$-open set,
$A_\alpha \subset Cl(\delta Int_I(A_\alpha)) \cup Int(\delta Cl_I(A_\alpha))$

Hence

$$\begin{aligned}\cup_\alpha A_\alpha &\subset \cup_\alpha[Cl(\delta Int_I(A_\alpha)) \cup Int(\delta Cl_I(A_\alpha))]\\ &\subset \cup_\alpha[Cl(\delta Int_I(A_\alpha))] \cup \cup_\alpha[Int(\delta Cl_I(A_\alpha))]\\ &\subset [Cl(\cup_\alpha(\delta Int_I(A_\alpha))] \cup [Int(\cup_\alpha(\delta Cl_I(A_\alpha))]\\ &\subset [Cl(\cup_\alpha(\delta Int_I(A_\alpha))] \cup [Int(\cup_\alpha(\delta Cl_I(A_\alpha))]\\ &\subset [Cl(\delta Int_I(\cup_\alpha A_\alpha))] \cup [Int(\delta Cl_I(\cup_\alpha A_\alpha))].\end{aligned}$$

$\cup_\alpha A_\alpha$ is e-$\mathcal{I}$-open. □

Example 2.16. *Let $X = \{a, b, c, d\}$ with a topology $\tau = \{\emptyset, X, \{a\}, \{b, d\}, \{a, b, d\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$. Then the set $A = \{a, c\}$ and $A = \{b, c\}$ are e-$\mathcal{I}$-open, but $A \cap B = \{c\}$ is not e-$\mathcal{I}$-open. Since $\{b, c\}$ and $\{b, c\} \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$. For $Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) = Cl(\emptyset) \cup Int(\{c, d\}) = Cl(\emptyset) \cup \emptyset = \emptyset \not\supseteq \{c\}$. So $A \cap B \not\subseteq Cl(\delta Int_I(A \cap B)) \cup Int(\delta Cl_I(A \cap B))$.*

Definition 2.17. *Let A be a subset of X.*

1. *The intersection of all e-$\mathcal{I}$-closed containing A is called the e-I-closure of A and its denoted by $Cl^*_e(A)$,*
2. *The e-$\mathcal{I}$-interior of A, denoted by $Int^*_e(A)$, is defined by the union of all e-I-open sets contained in A.*

Proposition 2.18. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then if $A \in EIO(X, \tau)$ and $B \in \tau^a$, then $A \cap B \in eO(X, \tau)$.*

Proof. Let $A \in EIO(X, \tau)$, i.e., $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A))$ and $B \in \tau^a$, i.e., $B \subset Int(Cl(\delta Int(B)))$. Then

$$\begin{aligned}A \cap B &\subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) \cap Int(Cl(\delta Int(B)))\\ &= [Cl(\delta Int_I(A)) \cap Int(Cl(\delta Int(B)))] \cup [Int(\delta Cl_I(A)) \cap Int(Cl(\delta Int(B)))]\\ &\subset [Cl(Cl(\delta Int_I(A))) \cap Cl(Cl(\delta Int(B)))] \cup [Int(\delta Cl_I(A)) \cap Cl(\delta Int(B))]\\ &\subset [Cl(Cl(\delta Int_I(A)) \cap Cl(\delta Int(B)))] \cup [Int(Cl(\delta Cl_I(A)) \cap Cl(\delta Int(B)))]\\ &\subset [Cl(Cl(\delta Int_I(A) \cap \delta Int(B))] \cup [Int(Cl(\delta Cl_I(A) \cap \delta Int(B)))]\\ &\subset [Cl(\delta Int_I(A \cap \delta Int(B)))] \cup [Int(\delta Cl_I(\delta Cl_I(A \cap B)))]\\ &\subset [Cl(\delta Int(A \cap B))] \cup [Int(\delta Cl(A \cap B))].\end{aligned}$$

Then $A \cap B \in eO(X, \tau)$. □

Remark 2.19.

1. *Let A be a subset of an ideal topological space $(X, \tau, \mathcal{I})$. Then A is e-$\mathcal{I}$-closed if and only if $Cl^*_e(A) = A$,*
2. *Let B be a subset of an ideal topological space $(X, \tau, \mathcal{I})$. Then B is e-$\mathcal{I}$-open if and only if $Int^*_e(B) = B$,*

Proposition 2.20. *Let A, B be a subsets of an ideal topological space $(X, \tau, \mathcal{I})$ such that A is e-$\mathcal{I}$-open and B is e-$\mathcal{I}$-closed in X. Then there exist e-$\mathcal{I}$-open set H and e-$\mathcal{I}$-closed set K such that $A \cap B \subset H$ and $K \subset A \cup B$.*

Proof. Let $K = Cl^*_e(A) \cap B$ and $H = A \cup Int^*_e(B)$. Then, K is e-$\mathcal{I}$-closed and H is e-$\mathcal{I}$-open. $A \subset Cl^*_e(A)$ implies $A \cap B \subset Cl^*_e(A) \cap B = K$ and $Int^*_e(B) \subset B$ implies $A \cup Int^*_e(B) = H \subset A \cup B$. □

Definition 2.21. 1. *A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is called e-dense if $Cl_e(S) = X$, where $Cl_e(S)$ [7] (Def 2.9) is the smallest e-closed sets containing S,*

2. *A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is called e-$\mathcal{I}$-dense if $Cl^*_e(S) = X$.*

3 strong $\mathcal{B}^*_I$-set

Definition 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. A subset A of X is called strong $\mathcal{B}^*_I$-set if $A = U \cap V$, where $U \in \tau$ and V is a strongly t-$\mathcal{I}$-set and $Int(\delta Cl_I(V)) = Cl(\delta Int_I(V))$.*

Proposition 3.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A be a subset of X. The following hold:*

1. *If A is strong $\mathcal{B}^*_I$-set, then A is a B_I-set,*

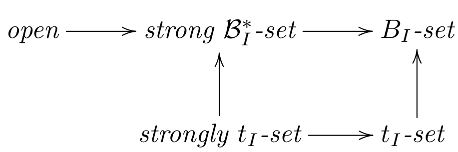
2. *If A is strongly t-$\mathcal{I}$-set, then A is a t-$\mathcal{I}$-set.*

Proof. 1. It follows from the fact every strongly t-$\mathcal{I}$-set is t-$\mathcal{I}$-set, the proof is obvious.

2. It follows from ([5] Theorem 21 (3)).

□

Remark 3.3. *The following diagram holds for a subset A of a space X:*



Remark 3.4. *The converses of proposition 3.2 (1), (2) need not to be true as the following examples show.*

Example 3.5. *Let* $X = \{a, b, c, d\}$ *with a topology* $\tau = \{\emptyset, X, \{a\}, \{a, c\}, \{a, b, c\}\}$ *and an ideal* $\mathcal{I} = \{\emptyset, \{c\}, \{a, c\}\}$. *Then the set* $A = \{a, c\}$ *is* B_I*-set, but not a strong* $\mathcal{B}_I^*$*-set and hence A is a* t_I*-set but not strongly t-*$\mathcal{I}$*-set. For* $Int(Cl^*(A)) = Int(\{a, c\}) = \{a\} = Int(A)$ *and hence A is a* t_I*-set. It is obvious that A is a* B_I*-set. But* $Int(\delta Cl_I(A)) = Int(\{X\}) = X$ *and* $Cl(\delta Int_I(A)) = Cl(\{a\}) = \{a, d\}$ *i.e* $Int(\delta Cl_I(A)) \neq Cl(\delta Int_I(A))$. *So A is not strong* $\mathcal{B}_I^*$*-set.*

Example 3.6. *Let* $X = \{a, b, c, d\}$ *with a topology* $\tau = \{\emptyset, X, \{b\}, \{b, c\}, \{b, c, d\}\}$ *and an ideal* $\mathcal{I} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. *Then the set* $A = \{b, c\}$ *is strong* $\mathcal{B}_I^*$*-set, but not a strongly t-*$\mathcal{I}$*-set.* $Int(\delta Cl_I(A)) = Int(\{X\}) = X$ *and* $Cl(\delta Int_I(A)) = Cl(\{b, c\}) = \{X\}$ *i.e* $Int(\delta Cl_I(A)) = Cl(\delta Int_I(A))$. *So A is strong* $\mathcal{B}_I^*$*-set. But,* $Int(\delta Cl_I(A)) = Int(\{X\}) = X \neq Int(A)$. *Therefor A is not a strongly t-*$\mathcal{I}$*-set.*

Proposition 3.7. *Let A be subset of an ideal topological space* $(X, \tau, \mathcal{I})$. *Then the following condition are equivalent:*

1. *A is open.*
2. *A is e-*$\mathcal{I}$*-open and strong* $\mathcal{B}_I^*$*-set.*

Proof. (1)⇒(2): By Remark 2.3 and Remark 3.3, every open set is e-$\mathcal{I}$-open. On other hand every open set is strongly $\mathcal{B}_I^*$-set.
(2)⇒(1): Let A is e-$\mathcal{I}$-open and strong $\mathcal{B}_I^*$-set. Then $A \subset Cl(\delta Int_I(A)) \cup Int(\delta Cl_I(A)) = Cl(\delta Int_I(U \cap V)) \cup Int(\delta Cl_I(U \cap V))$, where U is open and V is strongly t-$\mathcal{I}$-set and $Int(\delta Cl_I(V)) = Int(V)$, $Int(\delta Cl_I(V)) = Cl(\delta Int_I(V))$. Hence

$$\begin{aligned} A &\subset [Int(\delta Cl_I(U)) \cap Int(\delta Cl_I(V))] \cup [Cl(\delta Int_I(U)) \cap Cl(\delta Int_I(V))] \\ &= [U \cap Int(\delta Cl_I(V))] \cup [U \cap Cl(\delta Int_I(V))] \\ &\subset [U] \cap [Int(\delta Cl_I(V)) \cup Cl(\delta Int_I(V))] \\ &\subset [U] \cup [Int(\delta Cl_I(V)) \cap Int(\delta Int_I(V))] \\ &\subset [U] \cup [Int(\delta Cl_I(V))] \\ &\subset U \cup Int(V) = Int(A). \end{aligned}$$

On other hand, we have $U \cap Int(V) \subset U \cap V = A$. Thus, $A = U \cap Int(V)$ and A is open. □

4 decomposition of continuity

Definition 4.1. *[7] A function* $f : (X, \tau) \longrightarrow (Y, \sigma)$ *is said to be e-continuous if for each open set V of* (Y, σ), $f^{-1}(V)$ *is e-open.*

Definition 4.2. *A function* $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ *is said to be e-*$\mathcal{I}$*-continuous (resp. pre***-*$\mathcal{I}$*-continuous [5], strong* $\mathcal{B}_I^*$*-continuous) if for each open set V of* (Y, σ), $f^{-1}(V)$ *is e-*$\mathcal{I}$*-open (resp. pre***-*$\mathcal{I}$*-open, strong* $\mathcal{B}_I^*$*-set) in* $(X, \tau, \mathcal{I})$.

Definition 4.3. *A function* $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ *is said to be semi***-*$\mathcal{I}$*-continuous if for each open set V of* (Y, σ), $f^{-1}(V)$ *is semi***-*$\mathcal{I}$*-open in* $(X, \tau, \mathcal{I})$.

Proposition 4.4. *If a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is semi*-$\mathcal{I}$-continuous (pre*-$\mathcal{I}$-continuous), then f is e-$\mathcal{I}$-continuous.*

Proof. This is immediate consequence of Proposition 2.2 (2) and (3). □

Proposition 4.5. *If a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is strong $\mathcal{B}_I^*$-continuous, then f is B_I-continuous*

Proof. This is immediate consequence of Proposition 3.2 (1). □

Theorem 4.6. *For a function $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$. Then the following properties are equivalent,*

1. *f is continuous.*
2. *f is e-$\mathcal{I}$-continuous and strong $\mathcal{B}_I^*$-continuous.*

Proof. This is immediate consequence of Proposition 3.7. □

5 e-$\mathcal{I}$- continuous mappings

Definition 5.1.
1. *A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called δ-almost-continuous if the inverse image of each open set in Y is δ-preopen set in X [15].*
2. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is called δ-semicontinuous if the inverse image of each open set in Y is δ-semiopen set in X [6].*
3. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is called be a-continuous if for each open set V of (Y, σ), $f^{-1}(V)$ is a-open [4].*
4. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is called $\delta\alpha$-$\mathcal{I}$-continuous if for each δ_I-open set V of (Y, σ), $f^{-1}(V)$ is $\delta\alpha$-$\mathcal{I}$-open [8].*

Definition 5.2. *[16] Let (X, τ) be topological space and $A \subseteq X$. Then the set $\cap \{U \in \tau : A \subset U\}$ is called the kernel of A and denoted by $Ker(A)$.*

Lemma 5.3. *[10] Let (X, τ) be topological space and $A \subseteq X$.*

1. *$x \in Ker(A)$ if and only if $A \cap F \neq \emptyset$ for any closed subset of X with $x \in F$,*
2. *$A \subset Ker(A)$ and $A = Ker(A)$ if A is open in X,*
3. *if $A \subset B$, then $Ker(A) \subset Ker(B)$.*

Definition 5.4. *Let N be a subset of a space $(X, \tau, \mathcal{I})$, and let $x \in X$. Then N is called e-$\mathcal{I}$-neighborhood of x, if there exist e-$\mathcal{I}$-open set U containing x such that $U \subset N$.*

Theorem 5.5. *The following statement are equivalent for a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$:*

1. *f is e-$\mathcal{I}$-continuous,*
2. *for each $x \in X$ and each open set V in Y with $f(x) \in V$, there exist e-$\mathcal{I}$-open set U containing x such that $f(U) \subset V$,*
3. *for each $x \in X$ and each open set V in Y with $f(x) \in V$, $f^{-1}(V)$ is e-$\mathcal{I}$-neighborhood of x,*
4. *for every subset A of X, $f(Int_e^*(A)) \subset Ker(f(A))$,*
5. *for every subset B of Y, $Int_e^*(f^{-1}(B)) \subset f^{-1}(Ker(B))$.*

Proof. (1)⇒(2): Let $x \in X$ and let V be an open set in Y such that $f(x) \in V$. Since f is e-$\mathcal{I}$-continuous, $f^{-1}(V)$ is e-$\mathcal{I}$-open. By butting $U = f^{-1}(V)$ which is containing x, we have $f(U) \subset V$.
(2)⇒(3): Let V be an open set in Y such that $f(x) \in V$. Then by (2) there exists a e-$\mathcal{I}$-open set U containing x such that $f(U) \subset V$. So $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is e-$\mathcal{I}$-neighborhood of x.
(3)⇒(1): Let V be an open set in Y such that $f(x) \in V$. Then by (3), $f^{-1}(V)$ is e-$\mathcal{I}$–neighborhood of x. Thus for each $x \in f^{-1}(V)$, there exists a e-$\mathcal{I}$-open set U_x containing x such that $x \in U_x \subset f^{-1}(V)$. Hence $f^{-1}(V) \subset \bigcup_{x\in f^{-1}(V)} U_x$ and so $f^{-1}(V) \in EIO(X, \tau)$.
(1)⇒(5): Let A be any subset of X. Suppose that $y \notin Ker(A)$. Then, by Lemma 5.3, there exists a closed subset F of Y such that $y \in F$ and $f(A) \cap F = \emptyset$. Thus we have $A \cap f^{-1}(F) = \emptyset$ and $(Int_e^*(A)) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(Int_e^*(A)) \cap (F) = \emptyset$ and $y \notin f(Int_e^*(A))$. This implies that $f(Int_e^*(A)) \subset Ker(f(A))$
(5)⇒(6): Let B be any subset of Y. By (5) and Lemma 5.3, we have $f(Int_e^*(f^{-1}(B))) \subset Ker(f(f^{-1}(B))) \subset Ker(B)$ and $Int_e^*(f^{-1}(B)) \subset f^{-1}(Ker(B))$.
(6)⇒(1): Let V be any subset of Y. By (6) and Lemma 5.3, we have $Int_e^*(f^{-1}(V)) \subset f^{-1}(Ker(V))$=$f^{-1}(V)$ and $Int_e^*(f^{-1}(V))$=$f^{-1}(V)$. This shows that $f^{-1}(V)$ is e-$\mathcal{I}$-open. □

The following examples show that e-$\mathcal{I}$-continuous functions do not need to be semi*-$\mathcal{I}$-continuous and pre*-$\mathcal{I}$-continuous, and e-continuous function does not need to be e-$\mathcal{I}$-continuous.

Example 5.6. *Let $X = Y = \{a, b, c, d\}$ be a topology space by setting $\tau = \sigma = \{\emptyset, X, \{a\}, \{d\}, \{a, d\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$ on X. Define a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ as follows $f(a) = f(c) = d$ and $f(b) = f(d) = b$. Then f is e-$\mathcal{I}$-continuous but it is not pre*-$\mathcal{I}$-continuous.*

Example 5.7. *Let $X = Y = \{a, b, c\}$ be a topology space by setting $\tau = \sigma = \{\emptyset, X, \{a, b\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$ on X. Define a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ as follows $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is e-$\mathcal{I}$-continuous but it is not semi*-$\mathcal{I}$-continuous.*

Example 5.8. *Let (X, τ) be the real line with the indiscrete topology and (Y, τ) the real line with the usual topology and $\mathcal{I} = \{\emptyset\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is e-continuous but not e-$\mathcal{I}$-continuous.*

Proposition 5.9. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \longrightarrow (Z, \rho)$ be two functions, where $\mathcal{I}$ and $\mathcal{J}$ are ideals on X and Y, respectively. Then $g \circ f$ is e-$\mathcal{I}$-continuous if f is e-$\mathcal{I}$-continuous and g is continuous.*

Proof. The proof is clear. □

Proposition 5.10. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be e-$\mathcal{I}$-continuous and $U \in \tau$. Then the restriction $f_{|U} : (X, \tau_{|U}, \mathcal{I}_{|U}) \longrightarrow (Y, \sigma)$ is e-$\mathcal{I}$-continuous.*

Proof. Let V be any open set of (Y, σ). Since f is e-$\mathcal{I}$-continuous, $f^{-1}(V) \in EIO(X, \tau)$ and by Lemma 2.11, $f_{|U}^{-1}(V) = f^{-1}(V) \cap U \in EIO(U, \mathcal{I}_{|U})$. This shows that $f_{|U} : (X, \tau_{|U}, \mathcal{I}_{|U}) \longrightarrow (Y, \sigma)$ is e-$\mathcal{I}$-continuous. □

Theorem 5.11. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a function and let $\{U_\alpha : \alpha \in \Delta\}$ be an open cover of X. If the the restriction function $f|U_\alpha$ is e-$\mathcal{I}$-continuous for each $\alpha \in \Delta$, then f is e-$\mathcal{I}$-continuous.*

Proof. The proof is similar to that of Theorem 5.10 □

Lemma 5.12. *[20] For any function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$, $f(\mathcal{I})$ is an ideal on Y.*

Definition 5.13. *[20, 21] A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be I-compact if for every τ-open cover $\{\omega_\alpha : \alpha \in \Delta\}$ of A, there exists a finite subset Δ_o of Δ such that $(X - \cup\{\omega_\alpha : \alpha \in \Delta\}) \in \mathcal{I}$.*

Definition 5.14. *An ideal topological space $(X, \tau, \mathcal{I})$ is said to be e-$\mathcal{I}$-compact if for every e-$\mathcal{I}$-open cover $\{\omega_\alpha : \alpha \in \Delta\}$ of X, there exists a finite subset Δ_o of Δ such that $(X - \cup\{\omega_\alpha : \alpha \in \Delta\}) \in \mathcal{I}$.*

Theorem 5.15. *The image of e-$\mathcal{I}$-compact space under e-$\mathcal{I}$-continuous surjective function is $f(\mathcal{I})$-compact.*

Proof. Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a e-$\mathcal{I}$-continuous surjection and $\{V_\alpha : \alpha \in \Delta\}$ be an open cover of Y. Then $\left\{f^{-1}(V_\alpha) : \alpha \in \Delta\right\}$ is a e-$\mathcal{I}$-open cover of X due to our assumption on f. Since X is e-$\mathcal{I}$-compact, then there exists a finite subset Δ_o of Δ such that $(X - \cup\left\{f^{-1}(V_\alpha) : \alpha \in \Delta_o\right\}) \in \mathcal{I}$. Therefore $(Y - \cup\{V_\alpha : \alpha \in \Delta_o\}) \in f(\mathcal{I})$, which shows that $(Y, \sigma, f(\mathcal{I}))$ is $f(\mathcal{I})$-compact. □

Theorem 5.16. *A e-$\mathcal{I}$-continuous image of an e-$\mathcal{I}$-connected space is connected.*

Proof. Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is e-$\mathcal{I}$-continuous function of e-$\mathcal{I}$–connected space X onto a topological space Y. If possible, let Y be disconnected. Let A and B form a disconnected set of Y. Then A and B are clopen and $Y = A \cup B$, where $A \cap B = \emptyset$. Since f is e-$\mathcal{I}$-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = \emptyset$, where $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty e-$\mathcal{I}$-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \emptyset$. Hence X is non e-$\mathcal{I}$-connected, which is contradiction. Therefore, Y is connected. □

Definition 5.17. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$ is called e-$\mathcal{J}$-open (resp., e-$\mathcal{J}$-closed) if for each $U \in \tau$ (resp., closed set M in X), $f(U)$ (resp., $f(M)$) is e-$\mathcal{J}$-open (resp., e-$\mathcal{J}$-closed)*

Remark 5.18. *Every e-$\mathcal{I}$-open (resp., e-$\mathcal{I}$-closed) function is e-open (resp., e-closed) and the converses are false in general.*

Example 5.19. *Let $X = \{a, b, c\}$ be a topology space by setting $\tau_1 = \{\emptyset, X, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}, \{b\}, \{a\}\}$ and an ideal $\mathcal{I} = \{\emptyset, \{a\}\}$. Then the identity function $f : (X, \tau_1) \longrightarrow (X, \tau_2, \mathcal{I})$ is e-open but not e-$\mathcal{I}$-open.*

Example 5.20. *Let $X = \{a, b, c\}$ be a topology space by setting $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}, \{b\}, \{c\}\}$ and an ideal $\mathcal{I} = \{\emptyset, \{c\}\}$. Defined function $f : (X, \tau_1) \longrightarrow (X, \tau_2, \mathcal{I})$ as follows:$f(a) = a$, $f(b) = f(c) = b$. Then f is e-closed but not e-$\mathcal{I}$-closed.*

Theorem 5.21. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$ is e-$\mathcal{J}$-open if and only if for each $x \in X$ and each neighborhood U of x, there exists $V \in EJO(Y, \sigma)$ containing $f(x)$ such that $V \subset f(U)$.*

Proof. Suppose that f is a e-$\mathcal{J}$-open function. For each $x \in X$ and each neighborhood U of x, there exists $U_o \in \tau$ such that $x \in U_o \subset U$. Since f is e-$\mathcal{J}$-open, $V = f(U_o) \in EJO(Y, \sigma)$ and $f(x) \in V \subset f(U)$. Conversely, let U be an open set of (X, τ). For each $x \in U$, there exists $V_x \in EJO(Y, \sigma)$ such that $f(x) \in V_x \subset f(U)$. Therefore we obtain $f(U) = \bigcup\{V_x : x \in U\}$ and hence by Proposition 2.7, $f(U) \in EJO(Y, \sigma)$. This shows that f is e-$\mathcal{J}$-open. □

Theorem 5.22. *A function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma, \mathcal{J})$ be e-$\mathcal{J}$-open (resp., e-$\mathcal{J}$-closed). If W is any subset of Y and F is a closed (resp., open) set of X containing $f^{-1}(W)$, then there exists e-$\mathcal{J}$-closed (resp., e-$\mathcal{J}$-open) subset H of Y containing W such that $f^{-1}(W) \subset F$.*

Proof. Suppose that f is e-$\mathcal{J}$-open function. Let W be any subset of Y and F a closed subset of X containing $f^{-1}(W)$. Then $X - F$ is open and since f is e-$\mathcal{J}$-open, $f(X - F)$ e-$\mathcal{J}$-open. Hence $H = Y - f(X - F)$ is e-$\mathcal{J}$-closed. It follows from $f^{-1}(W) \subset F$ that $W \subset H$. Moreover, we obtain $f^{-1}(H) \subset F$. For e-$\mathcal{J}$-closed function. □

Theorem 5.23. *For any objective function $f : (X, \tau) \longrightarrow (Y, \sigma, \mathcal{J})$, the following are equivalent:*

1. *$f^{-1} : (Y, \sigma, \mathcal{J}) \longrightarrow (X, \tau)$ is e-$\mathcal{J}$-continuous,*
2. *f is e-$\mathcal{J}$-open,*
3. *f is e-$\mathcal{J}$-closed,*

Proof. It is straightforward. □

Definition 5.24. *A space (X, τ) is called*

1. *e-space if every e-open set of X is open in X.*
2. *submaximal if every dense subset of X is open in X [17].*
3. *extremely disconnected if the closure of every open set of X is open in X [27].*

Corollary 5.25. *If a function $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ is continuous, then f is e-$\mathcal{I}$-continuous.*

Corollary 5.26. *If (X, τ) is extremely disconnected and submaximal, then for any ideal $\mathcal{I}$ on X, $P^*IO(X, \tau) = S^*IO(X, \tau) = \delta SO(X, \tau) = \delta PO(X, \tau) = \delta\alpha IO(X, \tau) = aO(X, \tau) = \tau$.*

Corollary 5.27. *If (X, τ) is e-space, then for any ideal $\mathcal{I}$ on X, $EIO(X, \tau) = eO(X, \tau) = P^*IO(X, \tau) = S^*IO(X, \tau) = \delta SO(X, \tau) = \delta PO(X, \tau) = \delta\alpha IO(X, \tau) = aO(X, \tau) = \tau$.*

Corollary 5.28. *Let $f : (X, \tau, \mathcal{I}) \longrightarrow (Y, \sigma)$ be a function and let (X, τ) be e-space, then the following are equivalent:*

1. *f is e-$\mathcal{I}$-continuous,*
2. *f is e-continuous,*
3. *f is pre*-$\mathcal{I}$-continuous,*
4. *f is δ-almostcontinuous,*
5. *f is semi*-$\mathcal{I}$-continuous,*
6. *f is δ-semicontinuous,*
7. *f is $\delta\alpha$-$\mathcal{I}$-continuous,*
8. *f is $\delta\alpha$-continuous,*
9. *f is continuous,*

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