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Starlikeness and convexity of certain integral operators defined by convolution

Jyoti Aggarwal and Rachana Mathur

ABSTRACT: We define two new general integral operators for certain analytic functions in the unit disc $\mathcal{U}$ and give some sufficient conditions for these integral operators on some subclasses of analytic functions.

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1 Introduction

Let $\mathcal{A}_p(n)$ denote the class of all functions of the form

$$f(z) = z^p + \sum_{k=p+n}^{\infty} a_k z^k \, (p, n \in N = \{1, 2, 3...\}) \,. \tag{1.1}$$

which is analytic in open unit disc $\mathcal{U} = \{z \in \mathbb{C} | |z| < 1\}$.
In particular, we set

$$\mathcal{A}_p(1) = \mathcal{A}_p, \mathcal{A}_1(1) = \mathcal{A}_1 := \mathcal{A}.$$

If $f \in \mathcal{A}_p(n)$ is given by (1.1) and $g \in \mathcal{A}_p(n)$ is given by

$$g(z) = z^p + \sum_{k=p+n}^{\infty} b_k z^k \, (p, n \in N = \{1, 2, 3...\}) \,. \tag{1.2}$$

then the Hadamard product (or convolution) $f * g$ of f and g is given by

$$(f * g)(z) = z^p + \sum_{k=p+n}^{\infty} a_k b_k z^k = (g * f)(z). \tag{1.3}$$

We observe that several known operators are deducible from the convolutions. That is, for various choices of g in (1.3), we obtain some interesting operators. For example, for functions $f \in \mathcal{A}_p(n)$ and the function g is defined by

$$g(z) = z^p + \sum_{k=p+n}^{\infty} \psi_{k,m}(\alpha, \lambda, l, p) z^k \quad (m \in N_0 = N \cup \{0\}) \tag{1.4}$$

where

$$\psi_{k,m}(\alpha, \lambda, l, p) = \left[\frac{\Gamma(k+1)\Gamma(p-\alpha+1)}{\Gamma(p+1)\Gamma(k-\alpha+1)} . \frac{p+\lambda(k-p)+l}{p+l} \right]^m .$$

The convolution (1.3) with the function g is defined by (1.4) gives an operator studied by Bulut ([1]).

$$(f * g)(z) = D^{m,\alpha}_{\lambda,l,p} f(z)$$

Using convolution we introduce the new classes $\mathcal{US}^p_g(\delta, \beta, b)$ and $\mathcal{UK}^p_g(\delta, \beta, b)$ as follows

Definition 1.1 *A functions $f \in \mathcal{A}_p(n)$ is in the class $\mathcal{US}^p_g(\delta, \beta, b)$ if and only if f satisfies*

$$\operatorname{Re}\left\{ p + \frac{1}{b}\left(\frac{z(f*g)'(z)}{(f*g)(z)} - p \right) \right\} > \delta \left| \frac{1}{b}\left(\frac{z(f*g)'(z)}{(f*g)(z)} - p \right) \right| + \beta, \tag{1.5}$$

where $z \in \mathcal{U}, b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$.

Definition 1.2 *A functions $f \in \mathcal{A}_p(n)$ is in the class $\mathcal{US}^p_g(\delta, \beta, b)$ if and only if f satisfies*

$$\operatorname{Re}\left\{ p + \frac{1}{b}\left(1 + \frac{z(f*g)''(z)}{(f*g)'(z)} - p \right) \right\} > \delta \left| \frac{1}{b}\left(1 + \frac{z(f*g)''(z)}{(f*g)'(z)} - p \right) \right| + \beta, \tag{1.6}$$

where $z \in \mathcal{U}, b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$.

Note that

$$f \in \mathcal{UK}^p_g(\delta, \beta, b) \Longleftrightarrow \frac{zf'(z)}{p} \in \mathcal{US}^p_g(\delta, \beta, b).$$

Remark 1.1 *(i) For $\delta = 0$, we have*

$$\mathcal{UK}^p_g(0, \beta, b) = \mathcal{K}^p_g(\beta, b)$$
$$\mathcal{US}^p_g(0, \beta, b) = \mathcal{S}^p_g(\beta, b)$$

(ii) For $\delta = 0$ and $\beta = 0$

$$\mathcal{UK}^p_g(0, 0, b) = \mathcal{K}^p_g(b)$$
$$\mathcal{US}^p_g(0, 0, b) = \mathcal{S}^p_g(b)$$

(iii) For $\delta = 0$, $\beta = 0$ and $b = 1$

$$\mathcal{UK}^p_g(0, 0, b) = \mathcal{K}^p_g$$
$$\mathcal{US}^p_g(0, 0, b) = \mathcal{S}^p_g$$

(iv) For $(f_j * g)(z) = D^{m,\alpha}_{\lambda,l,p} f_j(z)$, we have two classes $\mathcal{UK}^{m,j,p,n}_{\alpha,\lambda,l}(\delta_j, \beta_j, b)$ and $\mathcal{US}^{m,j,p,n}_{\alpha,\lambda,l}(\delta_j, \beta_j, b)$ which is introduced by Guney and Bulut [1].

Definition 1.3 *Let $\eta \in N, m=(m_1,...,m_\eta) \in N_0^\eta$ and $k=(k_1,...,k_\eta) \in R_+^\eta$. One defines the following general integral operators:*

$$\begin{aligned} \mathcal{I}_g^{p,\eta,m,k} &: \mathcal{A}_p(n)^\eta \to \mathcal{A}_p(n) \\ \mathcal{G}_g^{p,\eta,m,k} &: \mathcal{A}_p(n)^\eta \to \mathcal{A}_p(n) \end{aligned} \tag{1.7}$$

such that

$$\begin{aligned} \mathcal{I}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \prod_{j=1}^{\eta} \left(\frac{(f_j * g)(t)}{t^p}\right)^{k_j} dt, \\ \mathcal{G}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \prod_{j=1}^{\eta} \left(\frac{(f_j * g)'(t)}{pt^{p-1}}\right)^{k_j} dt, \end{aligned} \tag{1.8}$$

where $z \in \mathcal{U},\ f_j, g \in \mathcal{A}_p(n), 1 \le j \le \eta$.

Remark 1.2 *(i) For $\eta = 1, m_1 = m, k_1 = k$, and $f_1 = f$, we have the new two new integral operators*

$$\begin{aligned} \mathcal{I}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \left(\frac{(f_j * g)(t)}{t^p}\right)^{k_j} dt, \\ \mathcal{G}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \left(\frac{(f_j * g)'(t)}{pt^{p-1}}\right)^{k_j} dt, \end{aligned} \tag{1.9}$$

(ii) For $(f_j * g)(z) = D_{\lambda,l,p}^{m,\alpha} f_j(z)$, we have

$$\begin{aligned} \mathcal{I}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \prod_{j=1}^{\eta} \left(\frac{D_{\lambda,l,p}^{m,\alpha} f_j(t)}{t^p}\right)^{k_j} dt, \\ \mathcal{G}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \prod_{j=1}^{\eta} \left(\frac{D_{\lambda,l,p}^{m,\alpha} f_j(t)'(t)}{pt^{p-1}}\right)^{k_j} dt, \end{aligned} \tag{1.10}$$

These operator were introduced by Bulut [].
(iii) If we take $g(z) = z^p/(1-z)$, the we have

$$\begin{aligned} \mathcal{I}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \prod_{j=1}^{\eta} \left(\frac{(f_j)(t)}{t^p}\right)^{k_j} dt, \\ \mathcal{G}_g^{p,\eta,m,k}(z) &= \int_0^z pt^{p-1} \prod_{j=1}^{\eta} \left(\frac{(f_j)'(t)}{pt^{p-1}}\right)^{k_j} dt, \end{aligned} \tag{1.11}$$

These two operators were introduced by Frasin [3].

2 Sufficient Conditions for $\mathcal{I}_g^{p,\eta,m,k}(z)$

Theorem 2.1 *Let $\eta \in N, m=(m_1,...,m_\eta) \in N_0^\eta$ and $k=(k_1,...,k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \ge 0, 0 \le \beta < p$, and $f_j \in \mathcal{US}_g^p(\delta,\beta,b)$ for $1 \le j \le \eta$. If*

$$0 \le p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{2.1}$$

then the integral operator $\mathcal{I}_g^{p,\eta,m,k}(z)$ *, defined by* (1.8) *, is in the class* $\mathcal{K}_g^p(\tau,b)$ *where*

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

Proof. From the definition (1.8), we observe that $\mathcal{I}_g^{p,\eta,m,k}(z) \in \mathcal{A}_p(n)$. We can easy to see that

$$\left(\mathcal{I}^{p,\eta,m,k}(z)\right)' = pz^{p-1}\prod_{j=1}^{\eta}\left(\frac{(f_j * g)(z)}{z^p}\right)^{k_j}. \tag{2.2}$$

Differentiating (2.2) logarithmically and multiplying by 'z', we obtain

$$\frac{z\left(\mathcal{I}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{I}^{p,\eta,m,k}(z)\right)'} = p - 1 + \sum_{j=1}^{\eta} k_j\left(\frac{z\left((f_j * g)(z)\right)'}{(f_j * g)(z)} - p\right) \tag{2.3}$$

or equivalently

$$1 + \frac{z\left(\mathcal{I}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{I}^{p,\eta,m,k}(z)\right)'} - p = \sum_{j=1}^{\eta} k_j\left(\frac{z\left((f_j * g)(z)\right)'}{(f_j * g)(z)} - p\right) \tag{2.4}$$

Then, by multiplying (2.4) with '1/b', we have

$$\frac{1}{b}\left(1 + \frac{z\left(\mathcal{I}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{I}^{p,\eta,m,k}(z)\right)'} - p\right) = \sum_{j=1}^{\eta} k_j\frac{1}{b}\left(\frac{z\left((f_j * g)(z)\right)'}{(f_j * g)(z)} - p\right) \tag{2.5}$$

or

$$\begin{aligned} p+&\frac{1}{b}\left(1 + \frac{z\left(\mathcal{I}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{I}^{p,\eta,m,k}(z)\right)'} - p\right) \\ &= p + \sum_{j=1}^{\eta} k_j\frac{1}{b}\left(\frac{z\left((f_j * g)(z)\right)'}{(f_j * g)(z)} - p + p - p\sum_{j=1}^{\eta} k_j\right) \end{aligned} \tag{2.6}$$

Since $f_j \in \mathcal{US}_g^p(\delta_j, \beta_j, b)$ $(1 \leq j \leq \eta)$, we get

$$\begin{aligned} \mathrm{Re}&\left\{p + \frac{1}{b}\left(1 + \frac{z\left(\mathcal{I}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{I}^{p,\eta,m,k}(z)\right)'} - p\right)\right\} \\ &= p + \sum_{j=1}^{\eta} k_j \mathrm{Re}\left\{\frac{1}{b}\left(\frac{z\left((f_j * g)(z)\right)'}{(f_j * g)(z)} - p\right)\right\} + p - \sum_{j=1}^{\eta} pk_j \\ &> \sum_{j=1}^{\eta} k_j\delta_j\left|\frac{1}{b}\left(\frac{z((f_j * g)(z))'}{(f_j * g)(z)} - p\right)\right| + p + \sum_{j=1}^{\eta} k_j(\beta_j - p). \end{aligned} \tag{2.7}$$

Since

$$\sum_{j=1}^{\eta} k_j \delta_j \left| \frac{1}{b} \left(\frac{z((f_j * g)(z))'}{(f_j * g)(z)} - p \right) \right| > 0$$

because the integral operator $\mathcal{I}_g^{p,\eta,m,k}(z)$, defined by (1.8) , is in the class $\mathcal{K}_g^p(\tau, b)$ with

$$\tau = p + \sum_{j=1}^{\eta} k_j (\beta_j - p).$$

■

3 Sufficient Conditions for $\mathcal{G}_g^{p,\eta,m,k}(z)$

Theorem 3.1 *Let $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ and $k = (k_1, ..., k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$, and $f_j \in \mathcal{US}_g^p(\delta, \beta, b)$ for $1 \leq j \leq \eta$. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j (\beta_j - p) < p, \tag{3.1}$$

then the integral operator $\mathcal{G}_g^{p,\eta,m,k}(z)$, defined by (1.8) *, is in the class $\mathcal{K}_g^p(\tau, b)$ where*

$$\tau = p + \sum_{j=1}^{\eta} k_j (\beta_j - p).$$

Proof. From the definition (1.8), we observe that $\mathcal{I}_g^{p,\eta,m,k}(z) \in \mathcal{A}_p(n)$. We can easy to see that

$$\left(\mathcal{G}^{p,\eta,m,k}(z)\right)' = pz^{p-1} \prod_{j=1}^{\eta} \left(\frac{(f_j * g)'(z)}{pz^{p-1}} \right)^{k_j}. \tag{3.2}$$

Differentiating (3.2) logarithmically and multiplying by 'z', we obtain

$$\frac{z\left(\mathcal{G}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{G}^{p,\eta,m,k}(z)\right)'} = p - 1 + \sum_{j=1}^{\eta} k_j \left(\frac{z\left((f_j * g)(z)\right)''}{(f_j * g)'(z)} + 1 - p \right) \tag{3.3}$$

or equivalently

$$1 + \frac{z\left(\mathcal{G}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{G}^{p,\eta,m,k}(z)\right)'} - p = \sum_{j=1}^{\eta} k_j \left(\frac{z\left((f_j * g)(z)\right)''}{((f_j * g)(z))'} + 1 - p \right) \tag{3.4}$$

Then, by multiplying (3.4) with '1/b', we have

$$\frac{1}{b} \left(1 + \frac{z\left(\mathcal{G}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{G}^{p,\eta,m,k}(z)\right)'} - p \right) = \sum_{j=1}^{\eta} k_j \frac{1}{b} \left(\frac{z\left((f_j * g)(z)\right)''}{(f_j * g)'(z)} + 1 - p \right) \tag{3.5}$$

or

$$p+\frac{1}{b}\left(\frac{z\left(\mathcal{G}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{G}^{p,\eta,m,k}(z)\right)'}+1-p\right)=p+\sum_{j=1}^{\eta}k_j\frac{1}{b}\left(\frac{z\left((f_j*g)(z)\right)''}{(f_j*g)'(z)}+1-p+p-p\sum_{j=1}^{\eta}k_j\right) \tag{3.6}$$

Since $f_j \in \mathcal{UK}_g^p(\delta_j,\beta_j,b)$ $(1\leq j\leq \eta)$, we get

$$\mathrm{Re}\left\{p+\frac{1}{b}\left(1+\frac{z\left(\mathcal{G}^{p,\eta,m,k}(z)\right)''}{\left(\mathcal{G}^{p,\eta,m,k}(z)\right)'}-p\right)\right\} \tag{3.7}$$

$$=p+\sum_{j=1}^{\eta}k_j\mathrm{Re}\left\{\frac{1}{b}\left(\frac{z((f_j*g)(z))''}{(f_j*g)'(z)}+1-p\right)\right\}+p-\sum_{j=1}^{\eta}pk_j+p+\sum_{j=1}^{\eta}k_j(\beta_j-p).$$

$$>\sum_{j=1}^{\eta}k_j\delta_j\left|\frac{1}{b}\left(\frac{z((f_j*g)(z))''}{(f_j*g)'(z)}+1-p\right)\right|+p+\sum_{j=1}^{\eta}k_j(\beta_j-p).$$

Since

$$\sum_{j=1}^{\eta}k_j\delta_j\left|\frac{1}{b}\left(\frac{z((f_j*g)(z))''}{(f_j*g)'(z)}+1-p\right)\right|>0$$

because the integral operator $\mathcal{G}_g^{p,\eta,m,k}(z)$, defined by (1.8) , is in the class $\mathcal{K}_g^p(\tau,b)$ with

$$\tau=p+\sum_{j=1}^{\eta}k_j(\beta_j-p).$$

■

4 Corollaries and Consequences

For $\eta=1, m_1=m, k_1=k$, and $f_1=f$, we have

Corollary 4.1 *Let $\eta\in N, m\in N_0^\eta$ and $k\in R_+^\eta$. Also let $b\in\mathbb{C}-\{0\}, \delta\geq 0, 0\leq\beta<p$, and $f\in\mathcal{US}_g^p(\delta,\beta,b)$ for $1\leq j\leq\eta$. If*

$$0\leq p+k(\beta-p)<p, \tag{4.1}$$

then the integral operator $\mathcal{I}_g^{p,\eta,m,k}(z)$ is in the class $\mathcal{K}_g^p(\tau,b)$ where

$$\tau=p+k(\beta-p).$$

Corollary 4.2 *Let $\eta\in N, m\in N_0^\eta$ and $k\in R_+^\eta$. Also let $b\in\mathbb{C}-\{0\}, \delta\geq 0, 0\leq\beta<p$, and $f\in\mathcal{US}_g^p(\delta,\beta,b)$ for $1\leq j\leq\eta$. If*

$$0\leq p+k(\beta-p)<p, \tag{4.2}$$

then the integral operator $\mathcal{G}_g^{p,\eta,m,k}(z)$ is in the class $\mathcal{K}_g^p(\tau,b)$ where

$$\tau=p+k(\beta-p).$$

For $(f_j * g)(z) = D^{m,\alpha}_{\lambda,l,p} f_j(z)$, we have

Corollary 4.3 *Let $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ and $k = (k_1, ..., k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$, and $f_j \in \mathcal{US}^{m,j,p,n}_{\alpha,\lambda,l}(\delta_j, \beta_j, b)$ for $1 \leq j \leq \eta$. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{4.3}$$

then the integral operator $\mathcal{I}_{p,\eta,m,k}(z)$ is in the class $\mathcal{K}^{p,n}(\tau, b)$ where

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

Corollary 4.4 *Let $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ and $k = (k_1, ..., k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$, and $\mathcal{UK}^{m,j,p,n}_{\alpha,\lambda,l}(\delta_j, \beta_j, b)$ for $1 \leq j \leq \eta$. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{4.4}$$

then the integral operator $\mathcal{G}_{p,\eta,m,k}(z)$ is in the class $\mathcal{K}^{p,n}(\tau, b)$ where

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

which are known results obtained by Guney and Bulut [2].
Further, if put $p = 1$, we have

Corollary 4.5 *Let $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ and $k = (k_1, ..., k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < 1$, and $f_j \in \mathcal{US}^p_g(\delta, \beta, b)$ for $1 \leq j \leq \eta$. If*

$$0 \leq 1 + \sum_{j=1}^{\eta} k_j(\beta_j - 1) < 1, \tag{4.5}$$

then the integral operator $\mathcal{I}_g^{1,\eta,m,k}(z)$ is in the class $\mathcal{K}^1_g(\tau, b)$ where

$$\tau = 1 + \sum_{j=1}^{\eta} k_j(\beta_j - 1).$$

Corollary 4.6 *Let $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ and $k = (k_1, ..., k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < 1$, and $f_j \in \mathcal{US}^1_g(\delta, \beta, b)$ for $1 \leq j \leq \eta$. If*

$$0 \leq 1 + \sum_{j=1}^{\eta} k_j(\beta_j - 1) < 1, \tag{4.6}$$

then the integral operator $\mathcal{G}_g^{1,\eta,m,k}(z)$ *is in the class* $\mathcal{K}_g^1(\tau, b)$ *where*

$$\tau = 1 + \sum_{j=1}^{\eta} k_j(\beta_j - 1).$$

Upon setting $g(z) = z^p/(1-z)$, we have

Corollary 4.7 *Let* $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ *and* $k = (k_1, ..., k_\eta) \in R_+^\eta$*. Also let* $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$*, and* $f_j \in \mathcal{US}^p(\delta, \beta, b)$ *for* $1 \leq j \leq \eta$*. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{4.7}$$

then the integral operator $\mathcal{G}^{p,\eta,m,k}(z)$ *is in the class* $\mathcal{K}^p(\tau, b)$ *where*

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

Corollary 4.8 *Let* $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ *and* $k = (k_1, ..., k_\eta) \in R_+^\eta$*. Also let* $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$*, and* $f_j \in \mathcal{US}^p(\delta, \beta, b)$ *for* $1 \leq j \leq \eta$*. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{4.8}$$

then the integral operator $\mathcal{G}^{p,\eta,m,k}(z)$ *is in the class* $\mathcal{K}^p(\tau, b)$ *where*

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

Upon setting $g(z) = z^p/(1-z)$ and $\delta = 0$, we have

Corollary 4.9 *Let* $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ *and* $k = (k_1, ..., k_\eta) \in R_+^\eta$*. Also let* $b \in \mathbb{C} - \{0\}, 0 \leq \beta < p$*, and* $f_j \in \mathcal{US}^p(0, \beta, b)$ *for* $1 \leq j \leq \eta$*. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{4.9}$$

then the integral operator $\mathcal{G}^{p,\eta,m,k}(z)$ *is in the class* $\mathcal{K}^p(\tau, b)$ *where*

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

Corollary 4.10 *Let $\eta \in N, m = (m_1, ..., m_\eta) \in N_0^\eta$ and $k = (k_1, ..., k_\eta) \in R_+^\eta$. Also let $b \in \mathbb{C} - \{0\}, \delta \geq 0, 0 \leq \beta < p$, and $f_j \in \mathcal{US}^p(0, \beta, b)$ for $1 \leq j \leq \eta$. If*

$$0 \leq p + \sum_{j=1}^{\eta} k_j(\beta_j - p) < p, \tag{4.10}$$

then the integral operator $\mathcal{G}^{p,\eta,m,k}(z)$ is in the class $\mathcal{K}^p(\tau, b)$ where

$$\tau = p + \sum_{j=1}^{\eta} k_j(\beta_j - p).$$

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Jyoti Aggarwal - corresponding author
email: maths.jyoti86@gmail.com

Rachana Mathur
email: rachnamathur@rediffmail.com

Department of Mathematics,
Govt. Dungar (P.G.) College,
Bikaner, India

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