On the solutions of a class of nonlinear functional integral equations in space $C[0,a]$

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ABSTRACT: The principal aim of this paper is to give sufficient conditions for solvability of a class of some nonlinear functional integral equations in the space of continuous functions defined on interval $[0,a]$. The main tool used in our study is associated with the technique of measures of noncompactness. We give also some examples satisfying the conditions of our main theorem but not satisfying the conditions in [8].

1 Introduction

Nonlinear integral equations are an important part of nonlinear analysis. It is caused by the fact that this theory is frequently applicable in other branches of mathematics and mathemathical physics, engineering, economics, biology as well in describing problems connected with real world, [5]. The measure of noncompactness and theory of integral equations are rapidly developing with the help of tools in functional analysis, topology and fixed-point theory. Many articles in the field of functional integral equations give different conditions for the existence of the solutions of some nonlinear functional integral equations. A. Aghajani and Y. Jalilian in [1], J. Banaś and K. Sadarangani in [3], Zeqing Liu et al. in [11] and so on are some of these. The following equation has been considered in [6] :

$$x(t) = f(t, x(\alpha(t))) \int_0^1 u(t, s, x(s))ds,$$

for $t \in [0,1]$. K. Maleknejad et al. in [7] and [8] studied the existence of the solutions of the following equations

$$x(t) = f(t, x(\alpha(t))) \int_0^t u(t, s, x(s))ds, \ t \in [0,1]$$

and

$$x(t) = g(t, x(t)) + f\left(t, \int_0^t u(t, s, x(s))ds, x\left(\alpha(t)\right)\right), \ t \in [0, a],$$

respectively. Then, İ. Özdemir et al. dealt with the following equation in [9] and [10]

$$x(t) = g(t, x(\beta(t))) + f(t, x(\alpha(t))) \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, \ t \in [0, a].$$

In this paper, we consider the following nonlinear functional integral equation:

$$x(t) = g(t, x(\alpha(t))) + f\left(t, \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, x(\beta(t))\right) \tag{1}$$

for $t \in [0, a]$. Note that the mentioned equation has rather general form and contains as particular cases a lot of nonlinear integral equations of Volterra type.

In next section, we present some definitions and preliminaries results about the concept of measure of noncompactness. In final section, we give our main result concerning with the solvability of the integral equation (1) by applying Darbo fixed point theorem associated with the measure of noncompactness defined by J. Banaś and K. Goebel [2] and finally we present some examples to show that our result is applicable.

2   Notations, definitions and auxiliary facts

In this section, we give some notations, definitions and results which will be needed further on. Assume that $(E, \|.\|)$ is an infinite Banach space with zero element θ. We write $B(x, r)$ to denote the closed ball centered at x with radius r and especially, we write B_r instead of $B(\theta, r)$. If X is a subset of E then the symbols $\overline{X}$ and Conv X stand for the closure and the convex closure of X, respectively. Moreover, let $\mathfrak{M}_E$ indicates the family of all nonempty bounded subsets of E and $\mathfrak{N}_E$ indicates the its subfamily of all relatively compact sets. Finally, the standard algebraic operations on sets are denoted by λX and $X + Y$, respectively.

We use the following definition of the measure of noncompactness, given in [2].

Definition 1 *A mapping* $\mu : \mathfrak{M}_E \to \mathbb{R}_+ = [0, +\infty)$ *is said to be a measure of non-compactness in* E *if it satisfies the following conditions:*

1. The family $\ker \mu = \{X \in \mathfrak{M}_E : \mu(X) = 0\}$ is nonempty and $\ker \mu \subset \mathfrak{N}_E$.

2. $X \subset Y \Rightarrow \mu(X) \leq \mu(Y)$.

3. $\mu(X) = \mu(\overline{X}) = \mu(\text{Conv } X)$.

4. $\mu(\lambda X + (1-\lambda)Y) \leq \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [0,1]$.

5. If (X_n) is a sequence of closed sets from $\mathfrak{M}_E$ such that $X_{n+1} \subset X_n$ $(n = 1, 2, ...)$ and if $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection set $\cap_{n=1}^{\infty} X_n$ is nonempty.

Now, let us suppose that M is nonempty subset of a Banach space E and $T : M \to E$ is a continuous operator which transforms bounded sets onto bounded ones. We say that T satisfies the Darbo condition (with a constant $k \geq 0$) with respect to measure of noncompactness μ if for any bounded subset X of M the inequality

$$\mu(TX) \leq k\mu(X)$$

holds. If T satisfies the Darbo condition with $k < 1$, then it is said to be a contraction with respect to μ, [4]. Now, we introduce the following Darbo type fixed point theorem.

Theorem 2 *Let C be a nonempty, closed, bounded and convex subset of the Banach space E, μ be a measure of noncompactness defined in E and let $F : C \to C$ be a continuous mapping. Assume that there exists a constant $k \in [0,1)$ such that*

$$\mu(FX) \leq k\mu(X) \tag{2}$$

for any nonempty subset X of C. Then F has a fixed point in set C, [2].

As is known the family of all real valued and continuous functions defined on interval $[0, a]$ is a Banach space with the standart norm

$$\|x\| = \max\{|x(t)| : t \in [0, a]\}.$$

Let X be a fixed subset of $\mathfrak{M}_{C[0,a]}$. For $\varepsilon > 0$ and $x \in X$, by $\omega(x, \varepsilon)$ we denote the modulus of continuity of function x, i.e.,

$$\omega(x, \varepsilon) = \sup\{|x(t_1) - x(t_2)| : t_1, t_2 \in [0, a] \text{ and } |t_1 - t_2| \leq \varepsilon\}.$$

Furthermore let $\omega(X, \varepsilon)$ and $\omega_0(X)$ are defined by

$$\omega(X, \varepsilon) = \sup\{\omega(x, \varepsilon) : x \in X\},$$

and

$$\omega_0(X) = \lim_{\varepsilon \to 0} \omega(X, \varepsilon). \tag{3}$$

The authors have shown in [2] that function ω_0 is a measure of noncompactness in space $C[0, a]$.

3   The main result

First of all we write I to denote interval $[0, a]$ throughout this section. We study functional integral equation (1) with the following hypotheses.

(a) Functions $\alpha,\ \beta : I \to I$, $\varphi : I \to \mathbb{R}_+$ and $\gamma : [0, C] \to I$ are continuous.

(b) $g : I \times \mathbb{R} \to \mathbb{R}$ is continuous and there exists nonnegative constant k such that

$$|g(t, x_1) - g(t, x_2)| \leq k\, |x_1 - x_2|$$

for all $t \in I$ and $x_1, x_2 \in \mathbb{R}$.

(c) $f : I \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and there exist nonnegative constants l and q such that

$$
\begin{aligned}
|f(t, x_1, y) - f(t, x_2, y)| &\leq\ l\, |x_1 - x_2|\,, \\
|f(t, x, y_1) - f(t, x, y_2)| &\leq\ q\, |y_1 - y_2|
\end{aligned}
$$

for all $t \in I$ and $x_1, x_2, y_1, y_2, x, y \in \mathbb{R}$.

(d) $u : I \times [0, C] \times \mathbb{R} \to \mathbb{R}$ is continuous and there exist positive constants m, n and p such that
$$|u(t, s, x)| \leq m + n\, |x|^p$$

for all $t \in I$ and $s \in [0, C]\,,\ x \in \mathbb{R}$.

(e) The inequality
$$M + N + Cl(m + n) + k + q < 1$$

holds, where C, M and N are the positive constants such that $\varphi(t) \leq C$, $|g(t, 0)| \leq M$ and $|f(t, 0, 0)| \leq N$ for all $t \in I$.

Theorem 3 *Under assumptions* $(a) - (e)$ *Eq.(1) has at least one solution in space* $C\,[0, a]\,.$

Proof. We define the continuous function $h : [0, 1] \to \mathbb{R}$ such that

$$h(r) = (k + q - 1)r + Cnlr^p + Clm + M + N,$$

where p is the constant given in assumption **(d)**. Then $h(0) > 0$ and $h(1) < 0$ by assumption **(e)**. Continuity of h guarantees that there exists number $r_0 \in (0, 1)$ such that $h(r_0) = 0$. Now, we will prove that Eq.(1) has at least one solution $x = x(t)$ belonging to $B_{r_0} \subset C\,[0, a]\,.$ We define operator T by

$$(Tx)(t) = g(t, x(\alpha(t))) + f\left(t, \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, x\,(\beta(t))\right),\ \ x \in C\,[0, a]\,.$$

Using the conditions of Theorem 3, we infer that Tx is continuous on I. For any $x \in B_{r_0}$, we have

$$
\begin{aligned}
|(Tx)(t)| &= \left| g(t, x(\alpha(t))) + f\left(t, \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, x(\beta(t))\right) \right| \\
&\leq |g(t, x(\alpha(t))) - g(t, 0)| + |g(t, 0)| \\
&\quad + \left| f\left(t, \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, x(\beta(t))\right) - f(t, 0, x(\beta(t))) \right| \\
&\quad + |f(t, 0, x(\beta(t)))) - f(t, 0, 0)| + |f(t, 0, 0)| \\
&\leq k |x(\alpha(t))| + M + l \left| \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds \right| + q |x(\beta(t))| + N \\
&\leq k \|x\| + M + Cl(m + n \|x\|^p) + q \|x\| + N \\
&\leq kr_0 + M + Cl(m + n(r_0)^p) + qr_0 + N \\
&= h(r_0) + r_0 \\
&= r_0.
\end{aligned}
$$

This result shows that operator T transforms ball B_{r_0} into itself. Now, we will prove that operator $T : B_{r_0} \to B_{r_0}$ is continuous. To do this, consider $\varepsilon > 0$ and any $x, y \in B_{r_0}$ such that $\|x - y\| \leq \varepsilon$. Then, we obtain the following inequalities by taking into account the assumptions of Theorem 3.

$$
\begin{aligned}
&|(Tx)(t) - (Ty)(t)| \\
=\ & \left| g(t, x(\alpha(t))) + f\left(t, \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, x(\beta(t))\right) \right. \\
& \left. -g(t, y(\alpha(t))) - f\left(t, \int_0^{\varphi(t)} u(t, s, y(\gamma(s)))ds, y(\beta(t))\right) \right| \\
\leq\ & |g(t, x(\alpha(t))) - g(t, y(\alpha(t)))| \\
& + \left| f\left(t, \int_0^{\varphi(t)} u(t, s, x(\gamma(s)))ds, x(\beta(t))\right) - f\left(t, \int_0^{\varphi(t)} u(t, s, y(\gamma(s)))ds, x(\beta(t))\right) \right| \\
& + \left| f\left(t, \int_0^{\varphi(t)} u(t, s, y(\gamma(s)))ds, x(\beta(t))\right) - f\left(t, \int_0^{\varphi(t)} u(t, s, y(\gamma(s)))ds, y(\beta(t))\right) \right| \\
\leq\ & k |x(\alpha(t)) - y(\alpha(t))| + l \int_0^{\varphi(t)} |u(t, s, x(\gamma(s))) - u(t, s, y(\gamma(s)))| \, ds \\
& + q |x(\beta(t)) - y(\beta(t))| \\
\leq\ & (k + q) \|x - y\| + Cl\omega_{u_3}(I, \varepsilon) \\
\leq\ & (k + q)\varepsilon + Cl\omega_{u_3}(I, \varepsilon),
\end{aligned}
\tag{4}
$$

where

$$\omega_{u_3}(I,\varepsilon) = \sup\left\{|u(t,s,x) - u(t,s,y)| : t \in I, \ s \in [0,C], \ x,y \in [-r_0, r_0] \text{ and } |x - y| \leq \varepsilon\right\}.$$

On the other hand, from the uniform continuity of function $u = u(t,s,x)$ on set $I \times [0,C] \times [-r_0, r_0]$, we derive that $\omega_{u_3}(I,\varepsilon) \to 0$ as $\varepsilon \to 0$. Hence, estimate (4) proves that operator T is continuous on B_{r_0}. Moreover, we show that operator T satisfies (2) with respect to measure of noncompactness ω_0 given by (3). To do this, we choose a fixed arbitrary $\varepsilon > 0$. Let us consider $x \in X$ and $t_1, t_2 \in I$ with $|t_1 - t_2| \leq \varepsilon$, for any nonempty subset X of B_{r_0}. Then,

$$
\begin{aligned}
&|(Tx)(t_1) - (Tx)(t_2)| \\
=& \left| g(t_1, x(\alpha(t_1))) + f\left(t_1, \int_0^{\varphi(t_1)} u(t_1, s, x(\gamma(s)))ds, x(\beta(t_1))\right) \right. \\
& \left. -g(t_2, x(\alpha(t_2))) - f\left(t_2, \int_0^{\varphi(t_2)} u(t_2, s, x(\gamma(s)))ds, x(\beta(t_2))\right) \right| \\
\leq& \ |g(t_1, x(\alpha(t_1))) - g(t_2, x(\alpha(t_1)))| + |g(t_2, x(\alpha(t_1))) - g(t_2, x(\alpha(t_2)))| \\
& + \left| f\left(t_1, \int_0^{\varphi(t_1)} u(t_1, s, x(\gamma(s)))ds, x(\beta(t_1))\right) \right. \\
& \left. -f\left(t_1, \int_0^{\varphi(t_2)} u(t_1, s, x(\gamma(s)))ds, x(\beta(t_1))\right) \right| \\
& + \left| f\left(t_1, \int_0^{\varphi(t_2)} u(t_1, s, x(\gamma(s)))ds, x(\beta(t_1))\right) \right. \\
& \left. -f\left(t_1, \int_0^{\varphi(t_2)} u(t_2, s, x(\gamma(s)))ds, x(\beta(t_1))\right) \right| \\
& + \left| f\left(t_1, \int_0^{\varphi(t_2)} u(t_2, s, x(\gamma(s)))ds, x(\beta(t_1))\right) \right. \\
& \left. -f\left(t_1, \int_0^{\varphi(t_2)} u(t_2, s, x(\gamma(s)))ds, x(\beta(t_2))\right) \right| \\
& + \left| f\left(t_1, \int_0^{\varphi(t_2)} u(t_2, s, x(\gamma(s)))ds, x(\beta(t_2))\right) \right. \\
& \left. -f\left(t_2, \int_0^{\varphi(t_2)} u(t_2, s, x(\gamma(s)))ds, x(\beta(t_2))\right) \right|
\end{aligned}
$$

$$\leq \quad \omega_g(I,\varepsilon) + k\left|x(\alpha(t_1)) - x(\alpha(t_2))\right|$$

$$+l\left|\int_0^{\varphi(t_1)} u(t_1,s,x(\gamma(s)))ds - \int_0^{\varphi(t_2)} u(t_1,s,x(\gamma(s)))ds\right|$$

$$+l\int_0^{\varphi(t_2)} \left|u(t_1,s,x(\gamma(s))) - u(t_2,s,x(\gamma(s)))\right|ds + q\left|x(\beta(t_1)) - x(\beta(t_2))\right| \quad (5)$$

$$+\omega_f(I,\varepsilon)$$

$$\leq \quad \omega_g(I,\varepsilon) + k\omega(x,\omega(\alpha,\varepsilon)) + l\left|-\int_{\varphi(t_1)}^{\varphi(t_2)} u(t_1,s,x(\gamma(s)))ds\right| + Cl\omega_{u_1}(I,\varepsilon)$$

$$+q\omega(x,\omega(\beta,\varepsilon)) + \omega_f(I,\varepsilon)$$

$$\leq \quad \omega_g(I,\varepsilon) + k\omega(x,\omega(\alpha,\varepsilon)) + l\omega(\varphi,\varepsilon)\left(m + n\left(r_0\right)^p\right)$$

$$+Cl\omega_{u_1}(I,\varepsilon) + q\omega(x,\omega(\beta,\varepsilon)) + \omega_f(I,\varepsilon), \quad (6)$$

where

$$\omega_g(I,\varepsilon) \quad = \quad \sup\left\{\left|g(t,x) - g(t',x)\right| : t,t' \in I, \ x \in [-r_0,r_0] \text{ and } |t - t'| \leq \varepsilon\right\},$$

$$\omega_{u_1}(I,\varepsilon) \quad = \quad \sup\left\{\left|u(t,s,x) - u(t',s,x)\right| : \right.$$
$$\left. t,t' \in I, \ s \in [0,C], \ x \in [-r_0,r_0] \text{ and } |t - t'| \leq \varepsilon\right\},$$

$$\omega_f(I,\varepsilon) \quad = \quad \sup\left\{\left|f(t,s,x) - f(t',s,x)\right| : \right.$$
$$\left. t,t' \in I, \ s \in [-A,A], \ x \in [-r_0,r_0] \text{ and } |t - t'| \leq \varepsilon\right\}$$

and $A = C(m + n\left(r_0\right)^p)$. Also,

$$\omega(\alpha_i,\varepsilon) = \sup\left\{\left|\alpha_i(t) - \alpha_i(t')\right| : t,t' \in I \text{ and } |t - t'| \leq \varepsilon\right\},$$

for $i = 1,2,3,4$ such that $\alpha_1 = \alpha$, $\alpha_2 = \beta$, $\alpha_3 = \varphi$ and $\alpha_4 = x$. Thus, by using estimate (6) we get

$$\omega(TX,\varepsilon) \quad \leq \quad \omega_g(I,\varepsilon) + k\omega(X,\omega(\alpha,\varepsilon)) + l\omega(\varphi,\varepsilon)(m + n\left(r_0\right)^p)$$
$$+Cl\omega_{u_1}(I,\varepsilon) + q\omega(X,\omega(\beta,\varepsilon)) + \omega_f(I,\varepsilon). \quad (7)$$

Since functions α, β and φ are uniformly continuous on set I by condition (**a**), we deduce that $\omega(\alpha,\varepsilon) \to 0$, $\omega(\beta,\varepsilon) \to 0$ and $\omega(\varphi,\varepsilon) \to 0$ as $\varepsilon \to 0$. Similarly, we have $\omega_g(I,\varepsilon) \to 0$, $\omega_f(I,\varepsilon) \to 0$ and $\omega_{u_1}(I,\varepsilon) \to 0$ as $\varepsilon \to 0$ since the functions g, f and u are uniformly continuous on sets $I \times [-r_0,r_0]$, $I \times [-A,A] \times [-r_0,r_0]$ and $I \times [0,C] \times [-r_0,r_0]$, respectively. Hence, (7) yields that

$$\omega_0(TX) \leq (k + q)\omega_0(X).$$

Thus, since $k + q < 1$ from condition (**e**), we get that operator T is a contraction on ball B_{r_0} with respect to measure of noncompactness ω_0. Therefore, Theorem 2 gives that operator T has at least one fixed point in B_{r_0}. Consequently, nonlinear functional integral equation (1) has at least one continuous solution in $B_{r_0} \subset C[0,a]$. This step completes the proof of Theorem 3. ∎

4   Examples

In this section, we shall discuss some examples to illustrate the applicability of Theorem 3.

Example 4 *We examine the nonlinear functional integral equation having the form*

$$x(t) = \frac{2 + x(t^2)}{56 + t^3} + \frac{2^t + t^2}{21} + \frac{x(\sqrt{t}) + 1}{9 + t^4} + \frac{2}{10 + t} \int_0^t \frac{\cos t + \sqrt{|x(s^2)|}}{2 + \ln(t + 1) + s^2 t^3} ds, \quad (8)$$

for $t \in I = [0, 1]$. Put

$$\beta(t) = \sqrt{t}, \ \varphi(t) = t, \ \alpha(t) = t^2, \ \gamma(s) = s^2,$$

$$g(t, x) = \frac{2 + x}{56 + t^3}, \ u(t, s, x) = \frac{\cos t + \sqrt{|x|}}{2 + \ln(t + 1) + s^2 t^3},$$

$$f(t, v, z) = \frac{2^t + t^2}{21} + \frac{z + 1}{9 + t^4} + \frac{2v}{10 + t}$$

and

$$k = \frac{1}{56}, \ M = \frac{1}{28}, \ l = \frac{1}{5}, \ q = \frac{1}{9}, \ N = \frac{17}{70}, \ C = 1, \ m = n = p = \frac{1}{2}.$$

*It can be easily seen that conditions (**d**) and (**e**) are verified. On the other hand, it is easy to verify that the other assumptions of Theorem 3 hold. Therefore, Theorem 3 guarantees that Eq.(8) has at least one solution $x = x(t) \in C[0, 1]$.*

Example 5 *Let us consider the nonlinear functional integral equation of the form*

$$x(t) = g(t, x(t)) + f\left(t, \int_0^t u(t, s, x(s)) ds, x(\beta(t))\right), \quad (9)$$

where g, f, u and β are the functions in Example 4. Since the conditions of Theorem 3 hold, Eq.(9) has at least one solution $x = x(t) \in C[0, 1]$ from Theorem 3.
    Since

$$|u(t, s, x)| = \left|\frac{\cos t + \sqrt{|x|}}{2 + \ln(t + 1) + s^2 t^3}\right| \leq \frac{1}{2} + \frac{1}{2}|x|^{\frac{1}{2}}$$

for all $t, s \in [0, 1]$ and $x \in \mathbb{R}$, condition (H3) in [8] doesn't hold. Hence, the result presented in [8] is inapplicable to integral Eq.(9).

Example 6 *Consider the following nonlinear functional integral equation:*

$$x(t) = \frac{1 + x(\sqrt{t})}{32 + t} + \frac{\cos(\sqrt{1 + t^2})}{8} + \frac{x(t^2)}{8 + t^2}$$

$$+ \frac{4}{16 + t} \int_0^{t^2} \frac{\exp(-t) + x(s^2)}{1 + t^2 + s \sin^2(1 + x^2(s^2))} ds. \quad (10)$$

We will look for solvability of this equation in space $C[0,1]$. Put

$$\alpha(t) = \sqrt{t}, \ \varphi(t) = \beta(t) = t^2, \ \gamma(s) = s^2,$$

$$g(t,x) = \frac{1+x}{32+t}, \ u(t,s,x) = \frac{\exp(-t)+x}{1+t^2+s\sin^2(1+x^2)},$$

$$f(t,v,z) = \frac{\cos(\sqrt{1+t^2})}{8} + \frac{z}{8+t^2} + \frac{4v}{16+t}$$

and

$$k = M = \frac{1}{32}, \ l = \frac{1}{4}, \ q = N = \frac{1}{8}, \ C = m = n = p = 1.$$

*One can see easily that conditions (**d**) and (**e**) of Theorem 3 are verified. On the other hand, it is easy to verify that the other assumptions of Theorem 3 hold. Therefore, Theorem 3 guarantees that Eq.(10) has at least one solution $x = x(t) \in C[0,1]$.*

Example 7 *Let us consider the nonlinear functional integral equation given as*

$$x(t) = g(t, x(t)) + f\left(t, \int_0^t u(t, s, x(s))ds, x(\beta(t))\right), \tag{11}$$

where g, f, u and β are the functions in Example 6. It is clear that the conditions of Theorem 3 satisfy. So, Eq.(11) has at least one solution $x = x(t) \in C[0,1]$ by Theorem 3.

Since

$$\kappa = \frac{1}{4}, \ \lambda = \frac{1}{8}, \ a = n = 1$$

and $\kappa > \frac{1-\lambda}{2+2an}$ in condition $(H4)$, the result in [8] is inapplicable to integral Eq.(11).

Acknowledgment

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